**3rd Year Higher Level Junior Certificate Notes 2008**

**Solving Quadratic Equations Involving Fractions**

EXAMPLE: Solve the equation $\frac{2}{x-1}-\frac{1}{x+2}=\frac{1}{2}$

Use the following steps to solve questions involving fractions.

1. Identify a Common Denominator. This is the ‘stuff’ on the bottom multiplied together. In this example this is ***(x-1)(x+2)(2).***
2. We take this common denominator and put it outside each of the three terms in the question. We will then cancel terms which are common on top and bottom and then multiply out what's left.

So, we get $\frac{\left(x-1\right)\left(x+2\right)\left(2\right)2}{(x-1)}$ – $\frac{\left(x-1\right)\left(x+2\right)\left(2\right)(1)}{(x+2)} $= $\frac{\left(x-1\right)\left(x+2\right)\left(2\right)(1)}{2}$

1. We now cross out what’s common on top and bottom. This leaves us with

$\frac{\left(x-1\right)\left(x+2\right)\left(2\right)2}{(x-1)}$ – $\frac{\left(x-1\right)\left(x+2\right)\left(2\right)(1)}{(x+2)} $= $\frac{\left(x-1\right)\left(x+2\right)\left(2\right)(1)}{2}$

1. Rewrite what’s left and multiply out.

**NB** Take your time when doing the multiplications. **NEVER** multiply three brackets at the one time. Multiply the two easiest brackets together first and then on the next line multiply this answer by the last bracket. This is especially important when you have minus signs involved and when you have two brackets which involve *x*s and one which is a number bigger than 1.

Be careful when dealing with the minus sign just before the = sign. You MUST leave the minus sign until the next line and apply it separately. Multiply the brackets as normal first and then apply the minus sign, which will change all of the signs. So, multiply 2 by $(x-1)$which is $(2x-2$), when we apply the minus sign on the outside we get $-2x+2$

$\left(x+2\right)\left(2\right)2-\left(x-1\right)\left(2\right)=\left(x-1\right)(x+2)$

$$4\left(x+2\right)-\left(2x-2\right)=x\left(x+2\right)-1(x+2)$$

$$4x+8-2x+2=x^{2}+2x-1x-2$$

Tidy up by moving everything to RHS as the $x^{2} $is already there. We always write our equations in the form $ax^{2}+bx+c=0$.

So, we get $x^{2}+2x-1x-4x+2x-2-8-2=0$

Which becomes $x^{2}-1x-12=0$

1. **We now factorise this quadratic as we have always done which becomes**

$$\left(x-4\right)\left(x+3\right)=0$$

$$x-4=0 and x+3=0$$

$$x=4 and x=-3$$

Another Example: Page 46 Q21

**Solve the equation** $x^{2}+5x-14=0. $**Hence find the four values for *t* in the equation**

$$\left(t-\frac{8}{t}\right)^{2}+5\left(t-\frac{8}{t}\right)-14=0$$

Factorise the equation as normal, giving us

$$x^{2}+5x-14=0$$

$$\left(x+7\right)\left(x-2\right)=0$$

$$x=-7 and x=2$$

Now, comparing the second equation as we did last week, we see that we are substituting *x* with $\left(t-\frac{8}{t}\right)$**,** so, we let each *x* answer = to this, giving us

$$\left(t-\frac{8}{t}\right)=2$$

Multiply everything by ‘t’ to get rid of our fraction.

$$t\left(t\right)-t\left(\frac{8}{t}\right)=2\left(t\right)$$

$$t^{2}-8=2t$$

write in form $ax^{2}+bx+c=0$

$t^{2}-2t-8=0$ Factorise

$$\left(t-4\right)\left(t+2\right)=0$$

$$t=4 and t=-2$$

These are our second two values for t.

Therefore, our four values for *t* are -8, 1, 4 and -2

$$\left(t-\frac{8}{t}\right)=-7$$

Multiply everything by ‘t’ to get rid of our fraction.

$$t\left(t\right)-t\left(\frac{8}{t}\right)=-7\left(t\right)$$

$$t^{2}-8=-7t$$

write in form $ax^{2}+bx+c=0$

$t^{2}+7t-8=0$ Factorise

$$\left(t+8\right)\left(t-1\right)=0$$

$$t=-8 and t=1$$

These are our first two values for t – now do the same with the second value we had for *x* by doing the exact same thing again

**mm**

The Quadratic Formula

Factorise $3x^{2}-7x-5=0$

No matter what combinations of different possibilities you try out, this cannot be factorised by the method we’ve used up to now. So, when we cannot factorise like this we use a formula which you must know off by heart.

The solutions to the equation $ax^{2}+bx+c=0 $can be found using the following formula

$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$

IMPORTANT

a = the number in front of the $x^{2}$

b = the number in front of the $x$

c = the number on its own with no letters

**BE CAREFUL WITH MINUS SIGNS**

In our example

In our example then, a = 3, b = -7 and c = -5

Subbing these values into the formula we get

$$x=\frac{--7\pm \sqrt{(-7)^{2}-4\left(3\right)(-5)}}{2(3)}$$

$$x=\frac{7\pm \sqrt{49+60}}{6}$$

$$x=\frac{7\pm \sqrt{109}}{6}$$

$x=\frac{7\pm 10.440306}{6}$

2.90671

At this point we divide the sum into two parts. Firstly we add the two numbers using the + sign from the formula and secondly we subtract them using the – from the formula.

$$x=\frac{7+10.440306}{6}$$

$$x=\frac{17.440306}{6}$$

$$x=2.90671$$

$$x=\frac{7-10.440306}{6}$$

$$x=\frac{-3.440306}{6}$$

$$x=-0.0573384$$

 The best way of knowing whether we have to use the formula or not is if the question says to write your answer correct to a certain number of decimal places. Always use the formula in these cases.