Question 1

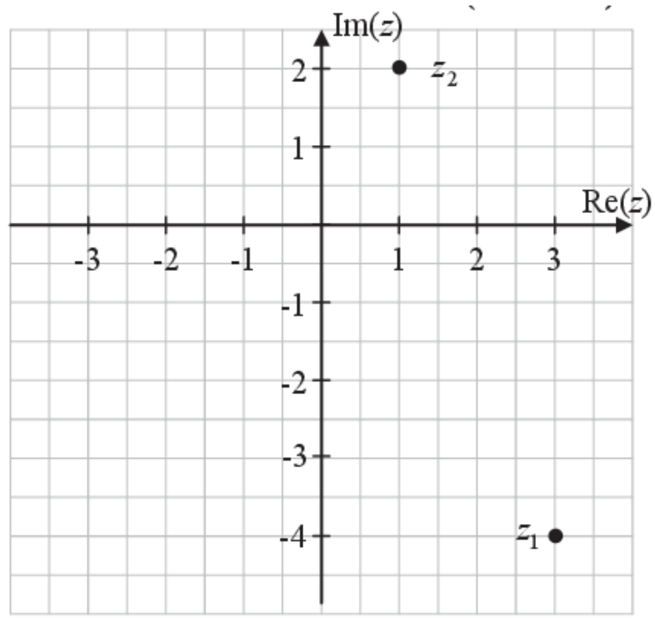
Let $z_1 = 3 - 4i$ and $z_2 = 1 + 2i$, where $i^2 = -1$.

(a) Plot z_1 and z_2 on the Argand diagram over.

(b) From your diagram, is it possible to say that $|z_1| > |z_2|$?
Give the reason for your answer.

Answer: Yes

Reason: The distance from the origin to z_1 is greater than the distance from the origin to z_2.



$$|z_1| = |3-4i| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$
$$|z_2| = |1+2i| = \sqrt{1^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$$
$$5 > \sqrt{5} \quad \Rightarrow \quad |z_1| > |z_2|$$

(d) Find $\frac{z_1}{z_2}$ in the form $x + yi$, where $x, y \in \mathbb{R}$.

$$\frac{z_1}{z_2} = \frac{3-4i}{1+2i} = \frac{3-4i}{1+2i} \times \frac{1-2i}{1-2i} = \frac{3-6i-4i+8i^2}{1^2+2^2} = \frac{-5-10i}{5} = -1-2i$$

Question 2

$$F = P(1+i)^t$$
$$= 800(1+0\cdot015)^{15}$$
$$= 800(1\cdot015)^{15}$$
$$= 1000\cdot18...$$

€$1000\cdot18... >$ €1000

$$\frac{95}{0\cdot 8473} = 112\cdot 1208... \approx €112\cdot 12$$

Question 11

(a) Alan pays income tax at the rate of 20%. He has weekly tax credits of €63. How much income tax does he pay?

Total tax: $€510 \times 0{\cdot}2 = €102$

Tax paid: $€102 - €63 = €39$

$€193 \times 0{\cdot}02 = €3{\cdot}86$
$€115 \times 0{\cdot}04 = €4{\cdot}60$

$€510 - (€193 + €115) = €202$

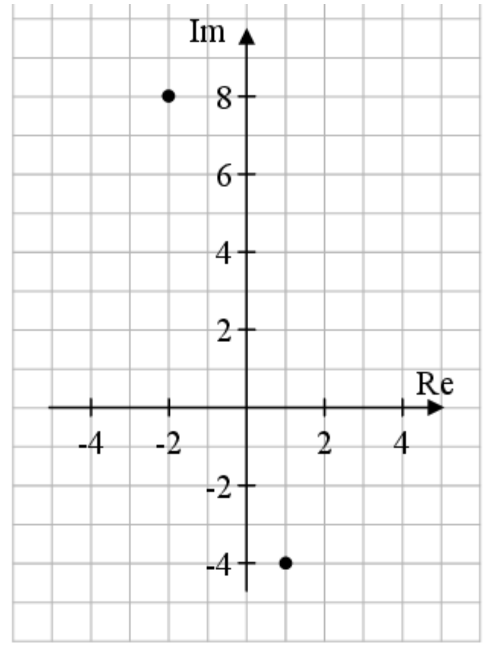
$€202 \times 0{\cdot}07 = €14{\cdot}14$

USC: $€3{\cdot}86 + €4{\cdot}60 + €14{\cdot}14 = €22{\cdot}60$

(c) Alan also pays PRSI. His total weekly deductions amount to €76·92. How much PRSI does Alan pay?

$\text{PRSI} = €76{\cdot}92 - (€39 + €22{\cdot}60) = €15{\cdot}32$

Question 12



(a) Plot z and $-2z$ on the Argand diagram.

$$-2z = -2(1-4i) = -2+8i$$

(b) Show that $2|z| = |-2z|$.

$$2|z| = 2|1-4i| = 2\sqrt{1^2+(-4)^2} = 2\sqrt{17}$$

$$|-2z| = |-2+8i| = \sqrt{(-2)^2+8^2} = \sqrt{68} = 2\sqrt{17}$$

$$\therefore 2|z| = |-2z|$$

(c) What does part **(b)** tell you about the points you plotted in part **(a)**?

$-2z$ is twice as far from the origin as z is.

(d) Let k be a real number such that $|z+k| = 5$. Find the two possible values of k.

$$|z+k| = 5 \quad \Rightarrow \quad |1-4i+k| = 5$$

$$\Rightarrow |(1+k)-4i| = 5 \Rightarrow \sqrt{(1+k)^2+(-4)^2} = 5$$

$$\Rightarrow (1+k)^2+16 = 25$$

$$\Rightarrow (1+k)^2 = 9 \;\Rightarrow\; 1+k = \pm 3 \;\Rightarrow\; k=2 \text{ or } k=-4$$

OR

$$(1+k)^2+16 = 25 \;\Rightarrow\; 1+2k+k^2+16-25 = 0 \;\Rightarrow\; k^2+2k-8 = 0$$

$$\Rightarrow (k-2)(k+4) = 0 \;\Rightarrow\; k=2 \text{ or } k=-4$$

Question 13

$$\tfrac{1}{2}(7x-2)+5=2x+7$$
$$\Rightarrow 7x-2+10=4x+14$$
$$\Rightarrow 7x+8=4x+14$$
$$\Rightarrow 3x=6$$
$$\Rightarrow x=2$$

$$\frac{2}{3x-4}-\frac{1}{2x+1}=\frac{1}{2}$$
$$\Rightarrow \frac{2(2x+1)(2)-1(3x-4)(2)=1(3x-4)(2x+1)}{(3x-4)(2x+1)(2)}$$
$$\Rightarrow 8x+4-6x+8=6x^2+3x-8x-4$$
$$\Rightarrow 6x^2-7x-16=0$$
$$\Rightarrow x=\frac{7\pm\sqrt{(-7)^2-4(6)(-16)}}{2(6)}=\frac{7\pm\sqrt{433}}{12}$$
$$\Rightarrow x=2{\cdot}3 \quad \text{or} \quad x=-1{\cdot}2$$

Question 14

BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG

(b) Peter says these outcomes are equally likely. Niamh says they are not. What do you need to know about the students in the school to decide which of them is correct?

The number of boys and the number of girls in the school.

(c) If all the outcomes are equally likely, what is the probability that the three students will be two girls followed by a boy?

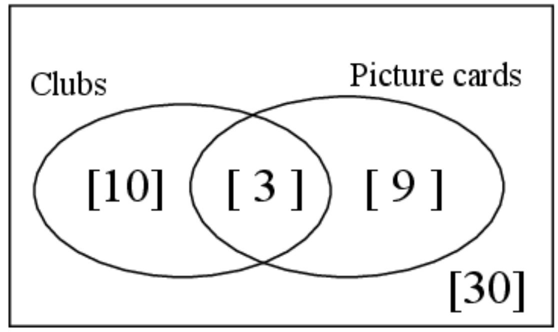
P(GGB) = $\frac{1}{8}$ or 0·125 or 12·5%

(d) Niamh guesses that there will be at least one girl among the next three students. Peter guesses that the next three students will be either three boys or two boys and a girl.
Who is more likely to be correct, assuming all outcomes are equally likely?
Justify your answer.

P(at least one girl) = $\frac{7}{8}$ or 0·875 or 87·5%
P(three boys or two boys and a girl = $\frac{4}{8}$ or $\frac{1}{2}$ or 0·5 or 50%
Niamh is more likely to be correct because of the greater probability.

Question 15

Show on the diagram the number of elements in each region.



(b) **(i)** A card is drawn from a pack of 52 cards.
Find the probability that the card drawn is the king of clubs.

P(king of clubs) $= \frac{1}{52}$

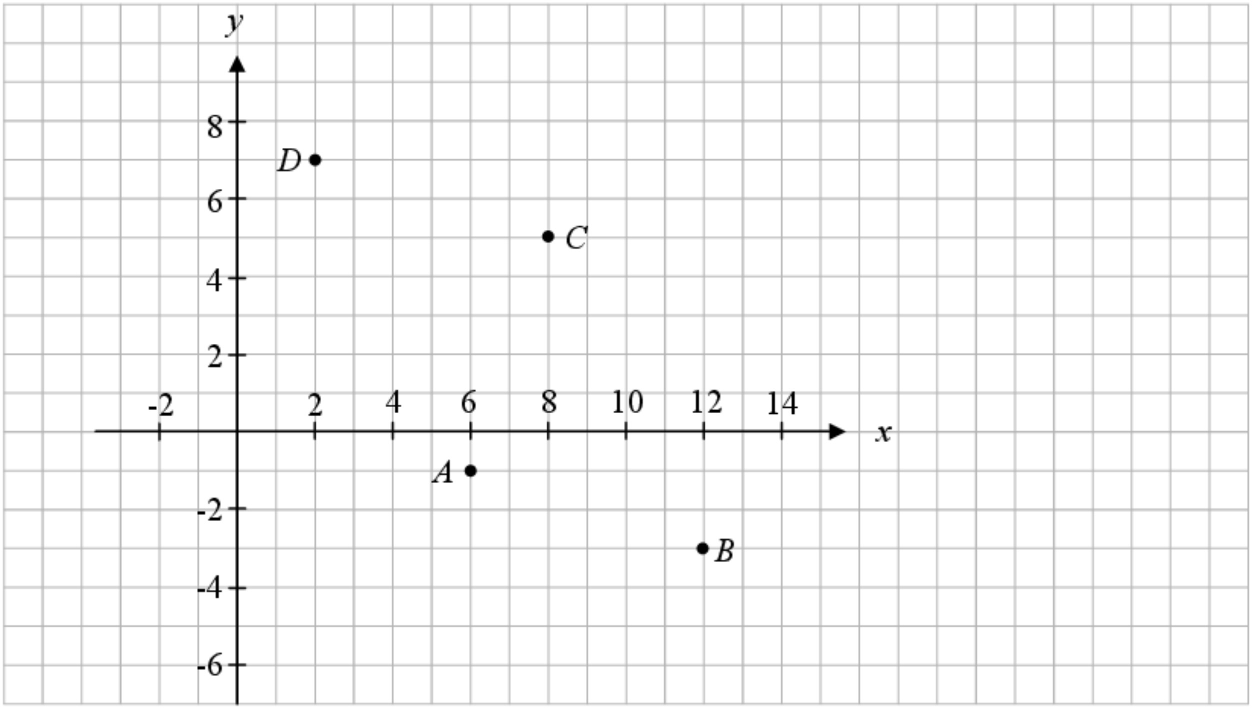
(ii) A card is drawn from a pack of 52 cards.
Find the probability that the card drawn is a club or a picture card.

P(club or picture card) $= \frac{22}{52} = \frac{11}{26}$

(iii) Two cards are drawn from a pack of 52 cards. Find the probability that neither of them is a club or a picture card. Give your answer correct to two decimal places.

P(not club or picture card) $= \frac{30}{52} \times \frac{29}{51} \approx 0{\cdot}33$

Question 16



(b) Describe two different ways of showing, using co-ordinate geometry techniques, that the points form a parallelogram *ABCD*.

Any TWO of:

- Show that opposite sides are parallel by showing the slopes of opposite lines are equal.
- Show that the diagonals bisect each other by showing the midpoint of $[AC]$ equals the midpoint of $[DB]$.
- Show that the opposite sides are equal in length using the length formula.
- Show that $\overrightarrow{AB}$ maps D onto C or similar.

(c) Use one of the ways you have described to show that *ABCD* is a parallelogram.

Slope $AB = \frac{-3+1}{12-6} = -\frac{2}{6}$, Slope $DC = \frac{7-5}{2-8} = -\frac{2}{6}$ $\Rightarrow AB \parallel DC$

Slope $BC = \frac{5+3}{8-12} = -2$, Slope $AD = \frac{7+1}{2-6} = -2$ $\Rightarrow BC \parallel AD$

Hence, *ABCD* a parallelogram

or

Midpoint $[AC] = \left(\frac{6+8}{2}, \frac{-1+5}{2}\right) = (7,2)$, Midpoint $[BD] = \left(\frac{12+2}{2}, \frac{-3+7}{2}\right) = (7,2)$,

$\Rightarrow$ Diagonals bisect. Hence, *ABCD* a parallelogram

or

Length $[AB] = \sqrt{(12-6)^2 + (-3+1)^2} = \sqrt{40}$

Length $[DC] = \sqrt{(2-8)^2 + (7-5)^2} = \sqrt{40}$

Length $[AD] = \sqrt{(6-2)^2 + (-1-7)^2} = \sqrt{80}$

Length $[BC] = \sqrt{(12-8)^2 + (-3-5)^2} = \sqrt{80}$

$\Rightarrow$ Opposite sides are equal. Hence, *ABCD* a parallelogram

or

$A(6,-1) \to B(12,-3)$ maps $D(2,7) \to (2+6, 7-2) = C(8,5)$ or similar

$\Rightarrow \overrightarrow{AB} = \overrightarrow{DC}$. Hence, *ABCD* a parallelogram

Question 17

Katie tossed a coin 200 times and threw 109 heads. Joe tossed the same coin 400 times and threw 238 heads. Lucy tossed the same coin 500 times and threw 291 heads. Katie, Joe and Lucy now think the coin may be biased.

(a) Give a reason why they think that the coin may be biased.

Each player tosses more than 50% heads.

(b) Lucy uses all the above data and calculates that the best estimate of the probability of throwing a head with this coin is 0·58. Show how Lucy might have calculated this probability.

Number of heads tossed: $109 + 238 + 291 = 638$

Total numbers of tosses: $200 + 400 + 500 = 1100$

$$\text{P(head)} = \frac{638}{1100} = 0 \cdot 58$$

(c) Joe agrees with Lucy's estimate of 0·58 as the probability of throwing a head with this coin. He claims that the probability of throwing 3 successive heads with this coin is **less** than the probability of throwing 2 successive tails. Calculate the probability of each event and state whether Joe's claim is true or not.

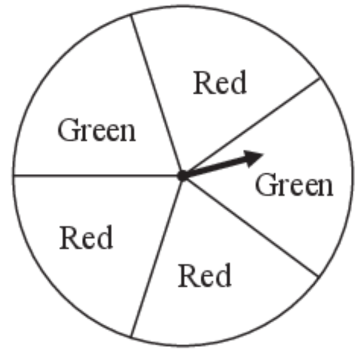
$$\text{P(3 heads)} = 0 \cdot 58^3 \approx 0 \cdot 195$$

$$\text{P(2 tails)} = 0 \cdot 42^2 \approx 0 \cdot 176$$

Joe's claim is not true.

Question 18

An unbiased circular spinner has a movable pointer and five equal sectors, two coloured green and three coloured red.



(a) **(i)** Find the probability that the pointer stops on green for one spin of the spinner.

$$P(\text{Green}) = \frac{2}{5}$$

(ii) List all the possible outcomes of three successive spins of the spinner.

RRR

RRG
RGR
GRR

RGG
GRG
GGR

GGG

(b) A game consists of spinning the spinner 3 times. Each time the spinner stops on green the player wins €1, otherwise the player wins nothing. For example, if the outcome of one game is "green, red, green" the player wins €2.

Complete the following table:

Player wins	€0	€1	€2	€3
Required outcomes	RRR	RRG RGR GRR	RGG GRG GGR	GGG

(c) Is one spin of the spinner above an example of a Bernoulli trial?

Answer: Yes

Explain what a Bernoulli trial is.

A Bernoulli trial is an experiment whose outcome is random and can be either of two possibilities: "success" or "failure".

Question 19

(a) l is the line $3x + 2y + 18 = 0$. Find the slope of l.

$$3x + 2y + 18 = 0 \quad \Rightarrow \quad 2y = -3x - 18 \quad \Rightarrow \quad y = -\tfrac{3}{2}x - 9$$

Slope $= -\tfrac{3}{2}$

(b) The line k is perpendicular to l and cuts the x-axis at the point $(7, 0)$.
Find the equation of k.

$$k \perp l \quad \Rightarrow \quad m \times -\tfrac{3}{2} = -1 \quad \Rightarrow \quad m = \tfrac{2}{3}$$

$$y - y_1 = m(x - x_1)$$
$$k : y - 0 = \tfrac{2}{3}(x - 7) \quad \Rightarrow \quad 3y = 2x - 14 \quad \Rightarrow \quad 2x - 3y - 14 = 0$$

(c) Find the co-ordinates of the point of intersection of the lines l and k.

$$\begin{array}{l} 3x + 2y = -18 \\ 2x - 3y = 14 \end{array} \quad \Rightarrow \quad \begin{array}{rl} 9x + 6y &= -54 \\ 4x - 6y &= 28 \\ \hline 13x \quad &= -26 \end{array} \quad \Rightarrow \quad x = -2$$

$$3x + 2y = -18 \quad \Rightarrow \quad 3(-2) + 2y = -18 \quad \Rightarrow \quad 2y = -12 \quad \Rightarrow \quad y = -6$$

Co-ordinates: $(-2, -6)$