## Question 1

## c(i)

Forming Equations

$$
\begin{array}{ll}
f(x)=4 x^{2}+b x+c & \\
f(2)=6 & f(-1)=0 \\
4(2)^{2}+b(2)+c=6 & 4(-1)^{2}+b(-1)+c=0 \\
16+2 b+c=6 & 4-b+c=0 \\
2 b+c+10=0 & -b+c=-4
\end{array}
$$

## Solving

$$
\begin{align*}
2 b+c & =-10 \\
-b+c & =-4 \\
-2 b-c & =10 \\
-b+c & =-4 \\
\hline-3 b & =6 \\
b & =-2
\end{aligned} \quad \begin{aligned}
b & =-2 \\
2 b+c & =-10 \\
2(-2)+c & =-10 \\
-4+c & =-10 \\
c & =-6
\end{align*}
$$

c(ii) 5 marks

Att 2

$$
\mathrm{c}(\mathrm{ii})
$$

$$
\begin{aligned}
& 4 x^{2}-2 x-6=-6 \\
& 4 x^{2}-2 x=0 \\
& 2 x(2 x-1)=0 \\
& 2 x-1=0 \quad \text { or } \quad 2 x=0 \\
& x=\frac{1}{2} \quad x=0
\end{aligned}
$$



|  |  |  |  |  | I |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & f(x)=5 x-4 \\ & f(0)=5(0)-4=0-4=-4 \rightarrow(0,-4) \\ & f(1)=5(1)-4=5-4=1 \rightarrow(1,1) \\ & f(2)=5(2)-4=10-4=6 \rightarrow(2,6) \\ & f(3)=5(3)-4=15-4=11 \rightarrow(3,11) \end{aligned}$ |  |  |  |  |  | $\begin{aligned} & g(x)=3 x+1 \\ & g(0)=3(0)+1=0+1=1 \rightarrow(0,1) \\ & g(1)=3(1)+1=3+1=4 \rightarrow(1,4) \\ & g(2)=3(2)+1=6+1=7 \rightarrow(2,7) \\ & g(3)=3(3)+1=9+1=10 \rightarrow(3,10) \end{aligned}$ |  |  |  |  |
|  |  |  |  |  | II |  |  |  |  |  |
| $x$ | 0 | 1 | 2 | 3 |  | $x$ | 0 | 1 | 2 | 3 |
| $5 x$ | 0 | 5 | 10 | 15 |  | $3 x$ | 0 | 3 | 6 | 9 |
| -4 | -4 | -4 | -4 | -4 |  | +1 | +1 | +1 | +1 | +1 |
| $f(x)$ | -4 | 1 | 6 | 11 |  | $g(x)$ | 1 | 4 | 7 | 10 |
| Points | $(0,-4)$ | $(1,1)$ | $(2,6)$ | $(3,11)$ |  | Points | $(0,1)$ | $(1,4)$ | $(2,7)$ | $(3,10)$ |

(b) (ii)

5 marks
Att 2
Point of intersection: $\quad(2.5,8.5)$

5 (c) (i) Draw the graph of $f$ for $-4 \leq x \leq 3, x \in \mathbf{R}$.

Part (c) (i)
10 marks
Att3


| I |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)=2 x^{2}+x-15$ |  |  |  |  |  |  |  |  |
| $f(-4)=2(-4)^{2}+(-4)-15=2(16)-4-15=32-19=13 \rightarrow(-4,13)$ |  |  |  |  |  |  |  |  |
| $f(-3)=2(-3)^{2}+(-3)-15=2(9)-3-15=18-18=0 \rightarrow(-3,0)$ |  |  |  |  |  |  |  |  |
| $f(-2)=2(-2)^{2}+(-2)-15=2(4)-2-15=8-17=-9 \rightarrow(-2,-9)$ |  |  |  |  |  |  |  |  |
| $f(-1)=2(-1)^{2}+(-1)-15=2(1)-1-15=2-16=-14 \rightarrow(-1,-14)$ |  |  |  |  |  |  |  |  |
| $f(0)=2(0)^{2}+(0)-15=(0)-0-15=0-15=-15 \rightarrow(0,-15)$ |  |  |  |  |  |  |  |  |
| $f(1)=2(1)^{2}+(1)-15=2(1)+1-15=2+1-15=3-15=-12 \rightarrow(1,-12)$ |  |  |  |  |  |  |  |  |
| $f(2)=2(2)^{2}+(2)-15=2(4)+2-15=8+2-15=10-15=-5 \rightarrow(2,-5)$ |  |  |  |  |  |  |  |  |
| $f(3)=2(3)^{2}+(3)-15=2(9)+3-15=18+3-15=21-15=6 \rightarrow(3,6)$ |  |  |  |  |  |  |  |  |
| II |  |  |  |  |  |  |  |  |
| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| $2(x)^{2}$ | 32 | 18 | 8 | 2 | 0 | 2 | 8 | 18 |
| $+x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| -15 | -15 | -15 | -15 | -15 | -15 | -15 | -15 | -15 |
| $f(x)$ | 13 | 0 | -9 | -14 | -15 | -12 | -5 | 6 |
| Points | $(-4,13)$ | $(-3,0)$ | (-2.-9) | $(-1,-14)$ | $(0,-15)$ | $(1,-12)$ | $(2,-5)$ | $(3,6)$ |

(ii)

Minimum value of $f(x):=-15 \cdot 1$ [indicated on graph]

(c) (iii)

5 marks
Att 2
(iii) $x \leq-3$ and $x \geq 21 / 2$ [indicated on graph]

## Question 3

(c) (i)

10 marks
Att 3

| $\begin{aligned} & f(0)=2 \\ & f(1)=2 \\ & f(2)=2 \end{aligned}$ | $\begin{aligned} & \rightarrow \mathbf{2 x} \\ & -1=-1 \\ & -1=1 \\ & -1=3 \end{aligned}$ |  |  |  | $\begin{aligned} & \rightarrow 4 x \\ = & 4(0)-4 \\ = & 4(1)-4 \\ = & 4(2)-4 \end{aligned}$ | (0.- $(1,0$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x$ | 0 | 1 | 2 | $x$ | 0 | 1 | 2 |
| $2 x$ | 0 | 2 | 4 | $4 x$ | 0 | 4 | 8 |
| -1 | -1 | -1 | -1 | -4 | -4 | -4 | -4 |
| $f(x)$ | -1 | 1 | 3 | $g(x)$ | -4 | 0 | 4 |
| Point | $(0,-1)$ | (1,1) | (2,3) |  | (0,-4) | $(1,0)$ | $(2,4)$ |


(ii) $\quad$ Point of intersection $=(1 \cdot 5,2) \quad$ tolerance $\pm 0.3$
(c) (iii)

5 marks
c(iii)

> I
> $y=2 x-1$
> $y=4 x-4$

$$
\begin{aligned}
& y=2 x-1 \\
& y=4 x-4 \\
& \quad \text { II } \\
& y=2 x-1 \quad \mathrm{x}-1 \\
& y=4 x-4
\end{aligned}
$$

$$
\begin{aligned}
4 x-4 & =2 x-1 \\
2 x & =3 \\
x & =1 \cdot 5
\end{aligned}
$$

$$
-y=-2 x+1
$$

$$
\frac{y=4 x-4}{0=2 x-3}
$$

$$
y=2(1 \cdot 5)-1
$$

$$
x=1 \cdot 5
$$

$$
y=3-1
$$

$$
y=2(1 \cdot 5)-1
$$

$$
y=2
$$

$$
y=3-1
$$

$$
y=2
$$

(c) (i)

$$
\begin{aligned}
& f(x)=x^{2}+b x+c \\
& f(-1)=(-1)^{2}+b(-1)+c=0 \quad \text { Eq } 1 \\
& 1-b+c=0 \\
& -b+c=-1 \\
& f(2)=(2)^{2}+b(2)+c=0 \quad \text { Eq } 2 \\
& 4+2 b+c=0 \\
& 2 b+c=-4 \\
& -b+c=-1 \\
& b-c=1 \\
& 2 b+c=-4 \\
& 3 b=-3 \\
& b=-1 \\
& 2(-1)+c=-4 \\
& -2+c=-4 \\
& c=-2
\end{aligned}
$$

c(ii)
(ii) $\quad f(x)=x^{2}-x-2$

$$
\begin{aligned}
& f(k)=k^{2}-k-2=-k+14 \\
& k^{2}-k-2=-k+14 \\
& k^{2}=16 \\
& k=4 \text { or } k=-4 \\
& \\
& \\
& \\
& \\
& \\
& (k-4)(k+4)=0 \\
& k=4 \text { or } k=-4
\end{aligned}
$$

## Question 5

Part (c)(i)
$(5,5)$ marks
Att 2,2
(i) $\quad f(-2)=1-3(-2)=7 \quad g(5)=1-(5)^{2}=1-25=-24$

Part (c)(ii) 5 marks
Att 2
(ii) $\quad f(x+1)=1-3(x+1)=1-3 x-3=-3 \mathrm{x}-2$

Part (c)(iii)

| (iii) $\quad$$-3 x-2=7+(-24)$ <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br> $-3 x=7-24+2$ <br> $x$$=5$ |  |
| :--- | :--- |
|  |  |

$$
\mathrm{f}(\mathrm{x})=5-3 \mathrm{x}-2 \mathrm{x}^{2}
$$

$$
\begin{array}{ll}
f(-3)=5-3(-3)-2(-3)^{2}=5+9-18=-4 & f(0)=5-3(0)-2(0)^{2}=5+0-0=5 \\
f(-2)=5-3(-2)-2(-2)^{2}=5+6-8=3 & f(1)=5-3(1)-2(1)^{2}=5-3-2=0 \\
f(-1)=5-3(-1)-2(-1)^{2}=5+3-2=6 & f(2)=5-3(2)-2(2)^{2}=5-6-8=-9
\end{array}
$$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| $-3 x$ | 9 | 6 | 3 | 0 | -3 | -6 |
| $-2 x^{2}$ | -18 | -8 | -2 | 0 | -2 | -8 |
| $f(x)$ | -4 | 3 | 6 | 5 | 0 | -9 |

(b) Linear Graph

5 marks
Att 2

$$
g(x)=-2 x-1
$$

$$
g(-3)=-2(-3)-1=6-1=5 \quad g(0)=-2(0)-1=0-1=-1
$$

$$
g(-2)=-2(-2)-1=4-1=3 \quad g(1)=-2(1)-1=-2-1=-3
$$

$$
g(-1)=-2(-1)-1=2-1=1 \quad g(2)=-2(2)-1=-4-1=-5
$$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $-2 x$ | 6 | 4 | 2 | 0 | -2 | -4 |
| -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $g(x)$ | 5 | 3 | 1 | -1 | -3 | -5 |

Graph of $f: x \rightarrow 5-3 x-2 x^{2}$ and $g: x \rightarrow-2 x-1$

(c) (i)

5 marks
Att 2
(i) Maximum value of $f(x)=6.1$
(c) (ii)

5 marks
Att 2
(ii) $x=1.5$ and $x=-2$
(c) (iii)

5 marks
Att 2
(iii) $f(x) \geq g(x) \quad \rightarrow \quad-2 \leq x \leq 1 \cdot 5$.

## Question 7

(c) $(\mathrm{i})$

15 marks
Att 5
(i) $\quad$ Cuts $x$ axis $\Rightarrow y=0$
$(-3)^{2}+b(-3)+c=0$
$9-3 b+c=0$
$-3 b+c=-9$
$(2)^{2}+b(2)+c=0$
$2 b+c=-4$
$4+2 b+c=0$
$3 b-c=9$
$5 b=5$
$b=1$ and $c=-6$
(c) (ii)
(ii) $x^{2}+x-6=(x+2)^{2}+(x+2)-6$

$$
x^{2}+x-6=x^{2}+4 x+4+x+2-6
$$

$$
x^{2}+x-6=x^{2}+5 x
$$

$$
-4 x=6
$$

$$
x=-1.5
$$

## Question 8


(b) Function $f$
$f: x \rightarrow x^{2}+5 x$

Att (3, 3)

| $x$ | -5 | -4 | -3 | -2 | -1 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $x^{2}$ | 25 | 16 | 9 | 4 | 1 | 0 | 1 |
| $+5 x$ | -25 | -20 | -15 | -10 | -5 | 0 | 5 |
| $f(x)$ | 0 | -4 | -6 | -6 | -4 | 0 | 6 |

(b) Function $g$
$g: x \rightarrow x+2$.

| $x$ | -5 | -4 | -3 | -2 | -1 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| +2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $g(x)$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |

(c) (i) 5 marks

Att 2
The minimum value of $f(x)=-6 \cdot 25$
(c) (ii) 5 marks

Att 2
$f(x)=g(x)$ at $x=0 \cdot 5$ and $x=-4 \cdot 5$
(c) (iii) 5 marks

Att 2
The range of values of $x$ for which $f(x) \leq g(x)$. $-4.5 \leq x \leq 0.5$

## Question 9


(c) (iii)

Solve

$$
\begin{array}{lc}
x^{2}-4 x+3=x-1 & \\
x^{2}-4 x+3-x+1=0 & \\
x^{2}-5 x+4=0 & \mathbf{2 m} \\
(x-4)(x-1)=0 & \\
x=1 \text { and } x=4 & \\
& \\
x^{2}-4 x+3=\mathrm{y} & \text { or } x-1=\mathrm{y} \\
(4)^{2}-4(4)+3=\mathrm{y} & 4-1=\mathrm{y} \\
16-16+3=3=\mathrm{y} & 3=\mathrm{y} \\
\quad \rightarrow \quad \text { point }(4,3) &
\end{array}
$$

II
Substitute $x=4$ into $f(x)=x^{2}-4 x+3$ and $g(x)=x-1$
$f(x)=x^{2}-4 x+3$
$f(x)=(4)^{2}-4(4)+3$
$f(x)=16-16+3$
$f(x)=3$
$(4,3)$
$g(x)=x-1$
$g(x)=4-1$
$g(x)=3$
$(4,3)$
$(4,3)$ on both lines $\rightarrow$ point of intersection

4m
5m


| $\boldsymbol{x}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $7 \boldsymbol{x}$ | $\mathbf{0}$ | 7 | $\mathbf{1 4}$ | $\mathbf{2 1}$ | $\mathbf{2 8}$ | $\mathbf{3 5}$ | $\mathbf{4 2}$ | $\mathbf{4 9}$ |
| $-\boldsymbol{x}^{2}$ | $\mathbf{0}$ | $-\mathbf{1}$ | $-\mathbf{4}$ | $-\mathbf{9}$ | $-\mathbf{1 6}$ | $-\mathbf{2 5}$ | $\mathbf{- 3 6}$ | $-\mathbf{4 9}$ |


| $\boldsymbol{x}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{f ( x )}$ | 0 | 6 | 10 | 12 | 12 | 10 | 6 | 0 |
| $\boldsymbol{f ( x )}$ | $\mathbf{0}$ | $\mathbf{6}$ | $\mathbf{1 0}$ | $\mathbf{1 2}$ | $\mathbf{1 2}$ | $\mathbf{1 0}$ | $\mathbf{6}$ | $\mathbf{0}$ |

or
or

| $f: x \rightarrow 7 x-x^{2}$. |
| :--- |
| $f: x \rightarrow 7(0)-0^{2}=0-0=0$ |
| $f: x \rightarrow 7(1)-1^{2}=7-1=6$ |
| $f: x \rightarrow 7(2)-2^{2}=14-4=10$ |
| $f: x \rightarrow 7(3)-3^{2}=21-9=12$ |
| $f: x \rightarrow 7(4)-4^{2}=28-16=12$ |
| $f: x \rightarrow 7(5)-5^{2}=35-25=10$ |
| $f: x \rightarrow 7(6)-6^{2}=42-36=6$ |
| $f: x \rightarrow 7(7)-7^{2}=49-49=0$ |
| $(0.0)$ |
| $(1,6)$ |
| $(2,10)$ |
| $(3,12)$ |
| $(4,12)$ |
| $(5,10)$ |
| $(6,6)$ |
| $(7,10)$ |

(c) (i)

5 marks
Att 2

Fills in any of the following points $(1,5)(2,8)(3,9)(4,8)(5,5)(6,0)$
or any other correct points to solve equation $a x-x^{2}=y$
I
e.g. $(1,5)$
$a(1)-(1)^{2}=5$
$a-1=5$
$a=5+1$
Value of $a=6$
II
$a x-x^{2}=y$
$x(a-x)=y$
Fill in e.g. $(1,5)$
$1(a-1)=5$
$a-1=5$
$a=5+1$
Value of $a=6$

