Question 1

Let $z_1 = 3 - 4i$ and $z_2 = 1 + 2i$, where $i^2 = -1$.

(a) Plot z_1 and z_2 on the Argand diagram over.

(b) From your diagram, is it possible to say that $|z_1| > |z_2|$?

Give the reason for your answer.

Answer:

Reason:
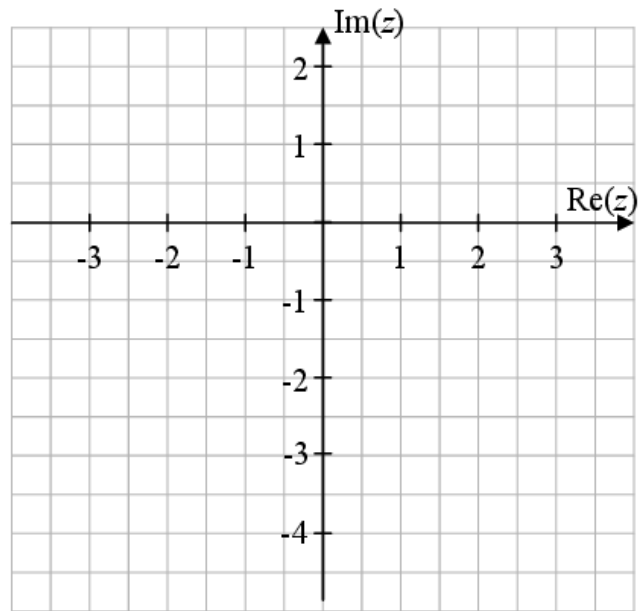
(c) Verify algebraically that $|z_1| > |z_2|$.

(d) Find $\dfrac{z_1}{z_2}$ in the form $x + yi$, where $x, y \in \mathbb{R}$.

Question 2

(b) Michael has a credit card with a credit limit of €1000. Interest is charged monthly at $1 \cdot 5\%$ of the amount owed. Michael gets a bill at the end of each month. At the start of January, Michael owes €800 on his credit card. If Michael makes no repayments and no more purchases, show that he will exceed his credit limit after 15 months.

(c) Michael buys an item costing £95 on the internet and pays with his credit card. If the exchange rate is €1 = £$0 \cdot 8473$, calculate, correct to the nearest cent, the amount that will be included on Michael's credit card bill.

Question 2   **(25 marks)**

The 2006 census shows that the number of males living in Ireland is about the same as the number of females.

(a)   If a person is selected at random, write down the probability that the person is male.

Answer: _____

(b)   Four people are chosen at random. We are interested in whether they are male or female.

   (i)   Complete the sample space below showing the sixteen equally likely outcomes.

   M M M M _____   _____   _____   _____

   M M M F _____   _____   _____   _____

   _____   _____   _____   _____

   _____   _____   _____   _____

   (ii)   Hence, or otherwise, complete the table of probabilities below.

four males	three males; one female	two males; two females	one male; three females	four females
$\dfrac{1}{16}$				

(c)   A person states the following: "If you pick four people at random, it's **more likely than not** that you'll get two males and two females."

Is this statement correct? Justify your answer using the answer(s) to part **(b)**.
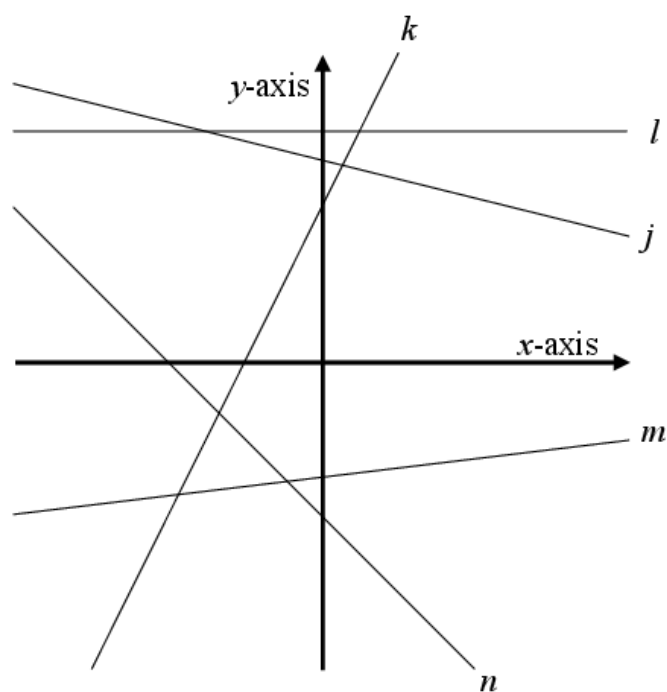
Answer: _____

Justification:

Question 4

Question 4 (25 marks)

(a) Five lines j, k, l, m, and n in the co-ordinate plane are shown in the diagram.

The slopes of the five lines are in the table below.

Complete the table, matching the lines to their slopes.

slope	line
2	
$\dfrac{1}{8}$	
0	
$-\dfrac{1}{4}$	
−1	

Question 5

Question 2   **(25 marks)**

(a)   A sum of €5000 is invested in an eight-year government bond with an annual equivalent rate (AER) of 6%. Find the value of the investment when it matures in eight years' time.

(b)   A different investment bond gives 20% interest after 8 years.
Calculate the AER for this bond.

Question 6

Question 3 (25 marks)

Two complex numbers are $u = 3 + 2i$ and $v = -1 + i$, where $i^2 = -1$.

(a) Given that $w = u - v - 2$, evaluate w.
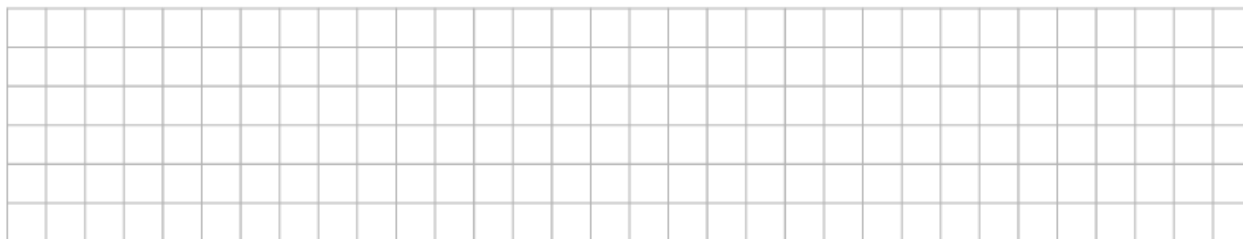
(b) Plot u, v, and w on the Argand diagram below.
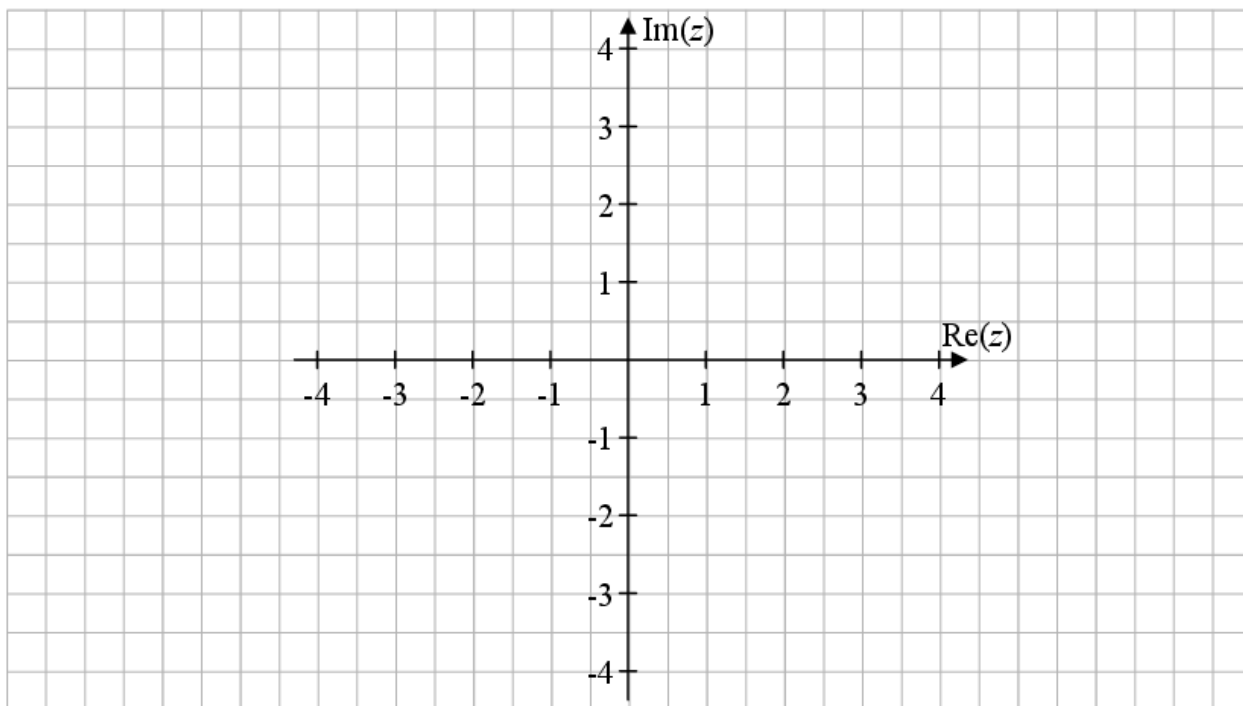
(c) Find $\dfrac{2u + v}{w}$.

Question 7

(25 marks)

(a) Solve the equation $x^2 - 6x - 23 = 0$, giving your answer in the form $a \pm b\sqrt{2}$, where $a, b \in \mathbb{Z}$.

(b) Solve the simultaneous equations

$$2r - s = 10$$

$$rs - s^2 = 12$$

Question 1 (25 marks)

(a)   State the *fundamental principle of counting*.

(b)   How many different ways are there to arrange five distinct objects in a row?

(c)   Peter is arranging books on a shelf. He has five novels and three poetry books. He wants to keep the five novels together and the three poetry books together. In how many different ways can he arrange the books?

Question 2 (25 marks)

A biased die is used in a game. The probabilities of getting the six different numbers on the die are shown in the table below.

Number	1	2	3	4	5	6
Probability	0·25	0·25	0·15	0·15	0·1	0·1

(a) Find the expected value of the random variable X, where X is the number thrown.

(b) There is a game at a funfair. It costs €3 to play the game. The player rolls a die once and wins back the number of euro shown on the die. The sentence below describes the difference between using the above biased die and using a fair (unbiased) die when playing this game. By doing the calculations required, complete the sentence.

"If you play the game many times with a fair die, you will win an average of _____ per game, but if you play with the biased die you will lose an average of _____ per game."
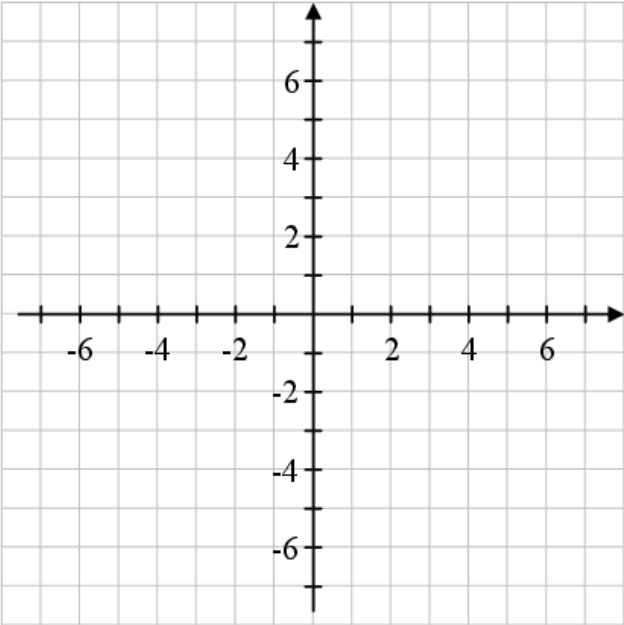
Question 3 **(25 marks)**

The points A, B, and C have co-ordinates as follows:

A (3, 5)

B (–6, 2)

C (4, –4)

(a) Plot A, B, and C on the diagram.

(b) Find the equation of the line AB.

(c) Find the area of the triangle ABC.

Question 1 (25 marks)

Alan pays income tax, a universal social charge (USC) and pay-related social insurance (PRSI) on his gross wages. His gross weekly wages are €510.

(a) Alan pays income tax at the rate of 20%. He has weekly tax credits of €63. How much income tax does he pay?

(b) Alan pays the USC at the rate of 2% on the first €193, 4% on the next €115 and 7% on the balance. Calculate the amount of USC Alan pays.
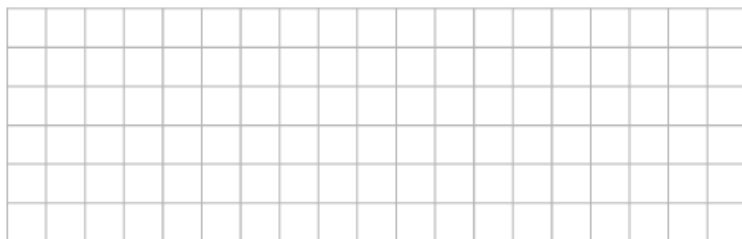
(c) Alan also pays PRSI. His total weekly deductions amount to €76·92. How much PRSI does Alan pay?
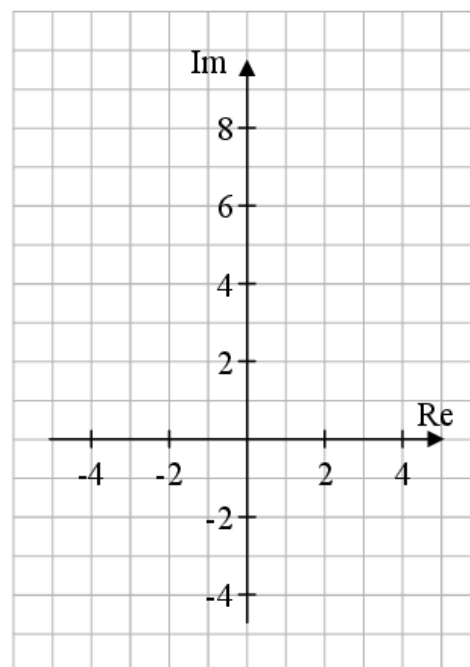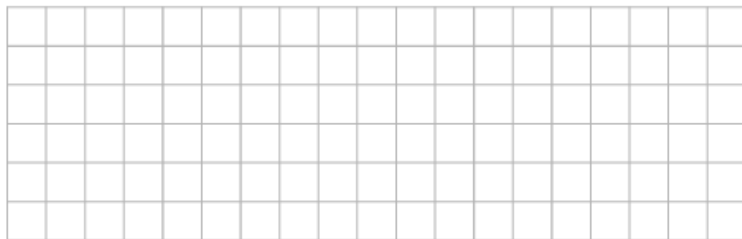
| page | running |

Question 12

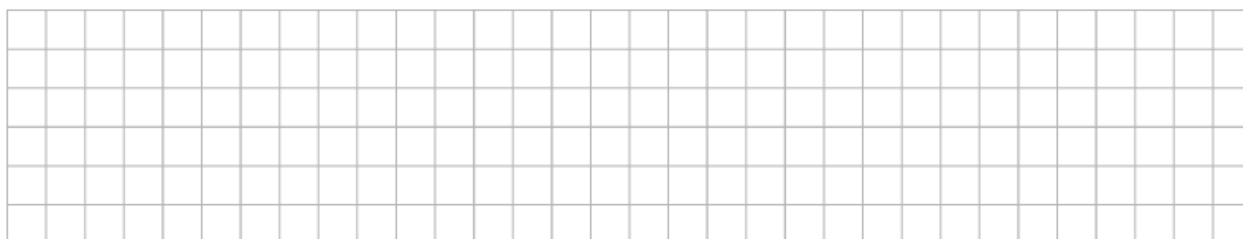The complex number $z = 1 - 4i$, where $i^2 = -1$.

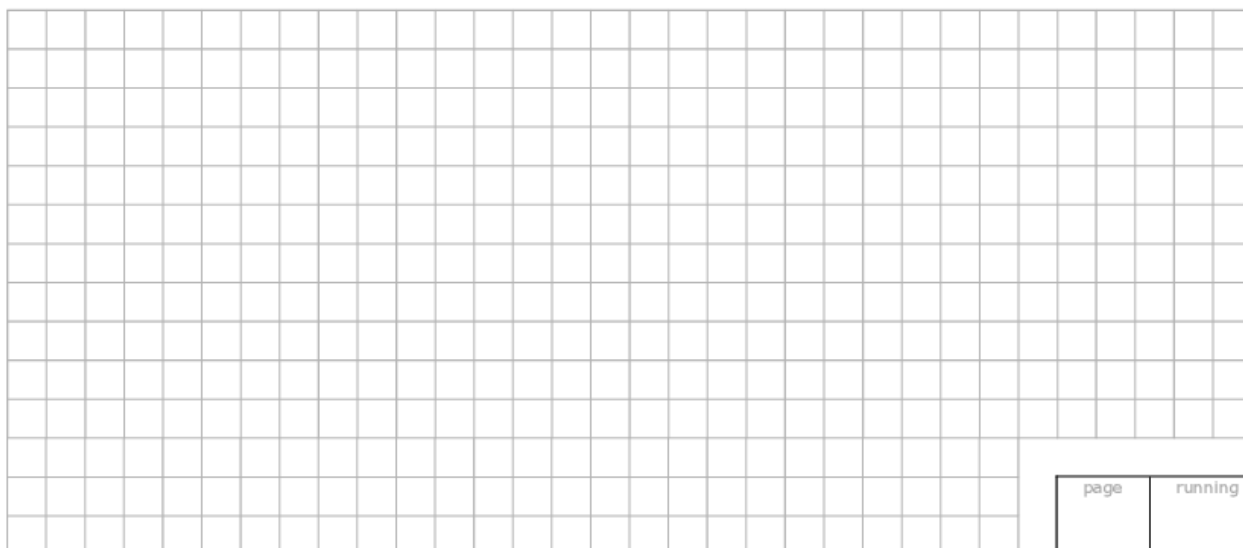**(a)** Plot z and $-2z$ on the Argand diagram.

(b) Show that $2|z| = |-2z|$.

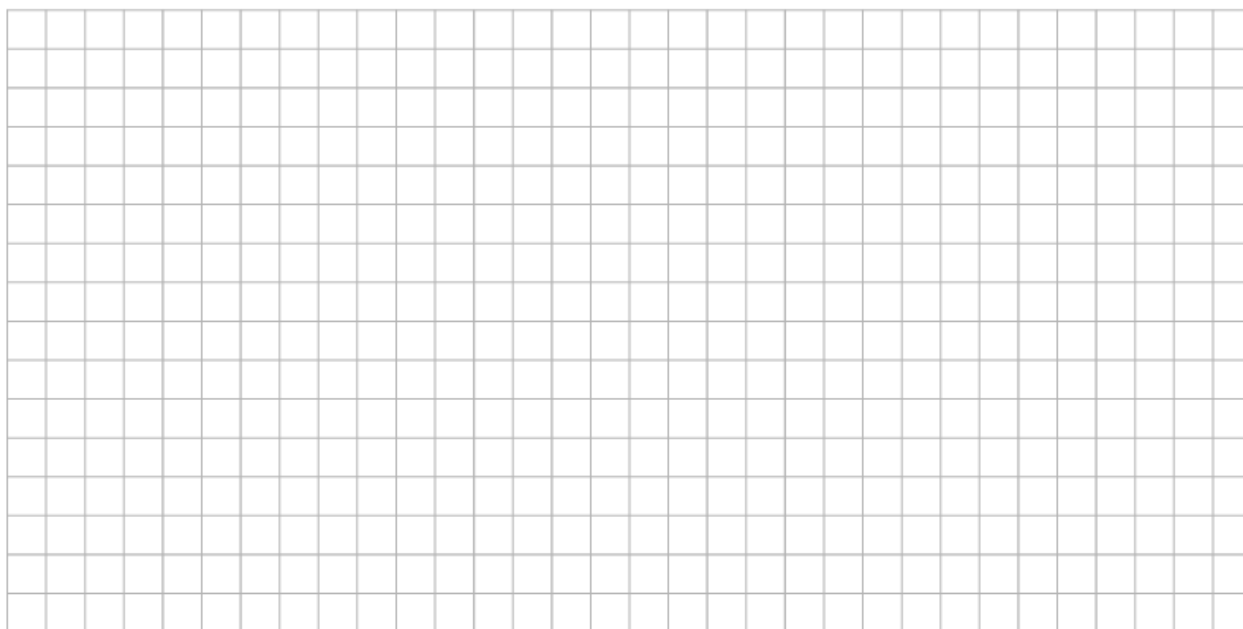**(c)** What does part **(b)** tell you about the points you plotted in part **(a)**?

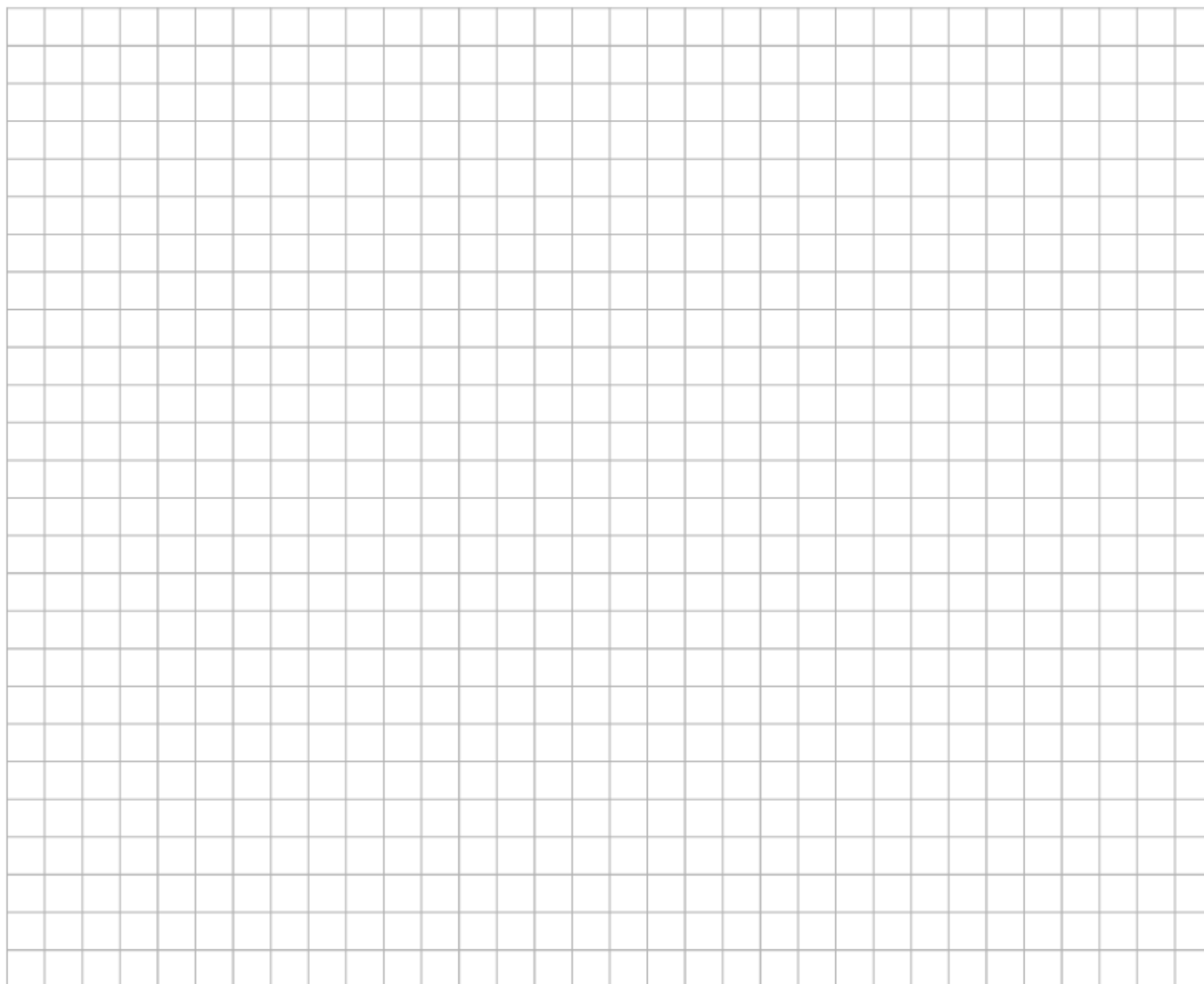**(d)** Let k be a real number such that $|z + k| = 5$. Find the two possible values of k.

Question 13

(a) Solve the equation $\frac{1}{2}(7x-2)+5=2x+7$.

(b) Solve the equation $\dfrac{2}{3x-4}-\dfrac{1}{2x+1}=\dfrac{1}{2}$ and give your answers correct to one decimal place.

Question 14

Peter and Niamh go to a large school. One morning, they arrive early. While they are waiting, they decide to guess whether each of the next three students to come in the door will be a boy or a girl.

(a) Write out the sample space showing all the possible outcomes. For example, BGG is one outcome, representing Boy, Girl, Girl.

(b) Peter says these outcomes are equally likely. Niamh says they are not. What do you need to know about the students in the school to decide which of them is correct?

(c) If all the outcomes are equally likely, what is the probability that the three students will be two girls followed by a boy?

(d) Niamh guesses that there will be at least one girl among the next three students. Peter guesses that the next three students will be either three boys or two boys and a girl.
Who is more likely to be correct, assuming all outcomes are equally likely?
Justify your answer.

Question 15

(a) In the Venn diagram below, the universal set is a normal deck of 52 playing cards. The two sets shown represent *clubs* and *picture cards* (kings, queens and jacks).

Show on the diagram the number of elements in each region.

Clubs   Picture cards

[   ]   ([   ])   [   ]

[   ]
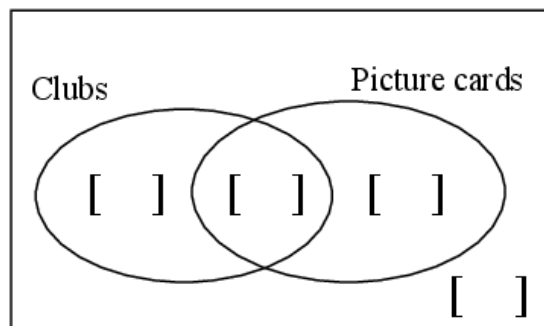
(b) **(i)** A card is drawn from a pack of 52 cards.
Find the probability that the card drawn is the king of clubs.

(ii) A card is drawn from a pack of 52 cards.
Find the probability that the card drawn is a club or a picture card.
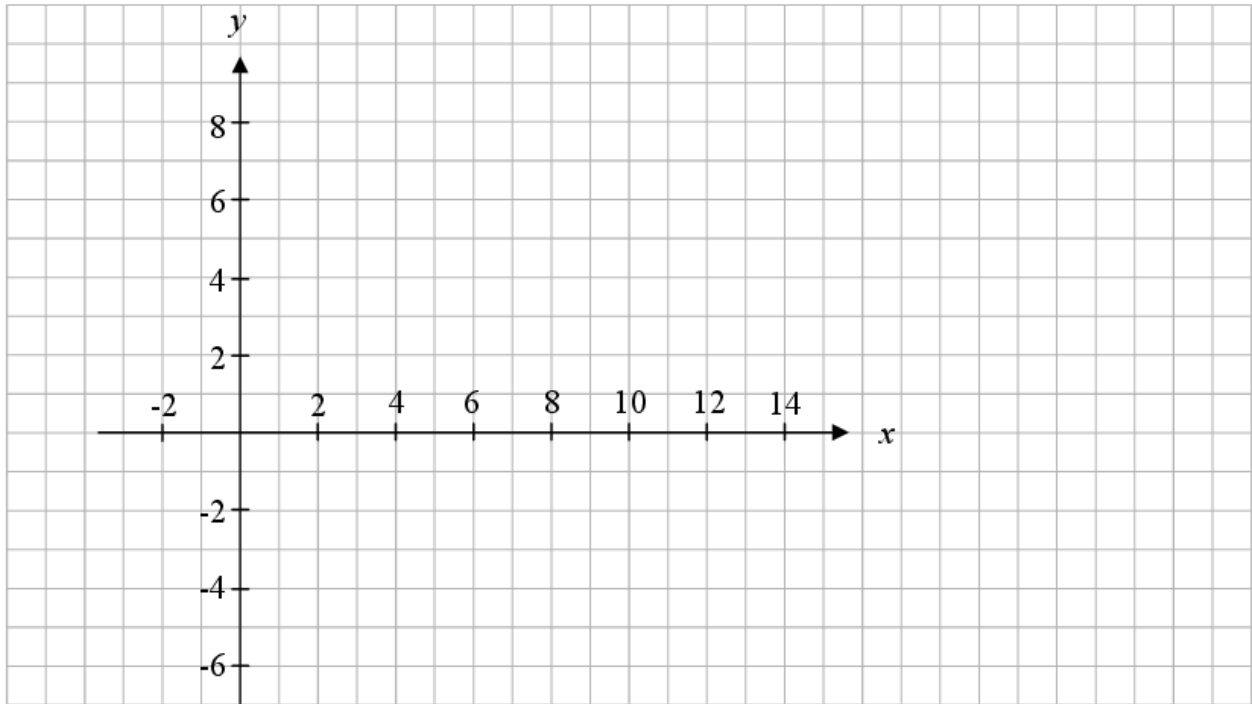
(iii) Two cards are drawn from a pack of 52 cards. Find the probability that neither of them is a club or a picture card. Give your answer correct to two decimal places.

Question 16

$A(6, -1), B(12, -3), C(8, 5)$ and $D(2, 7)$ are four points.
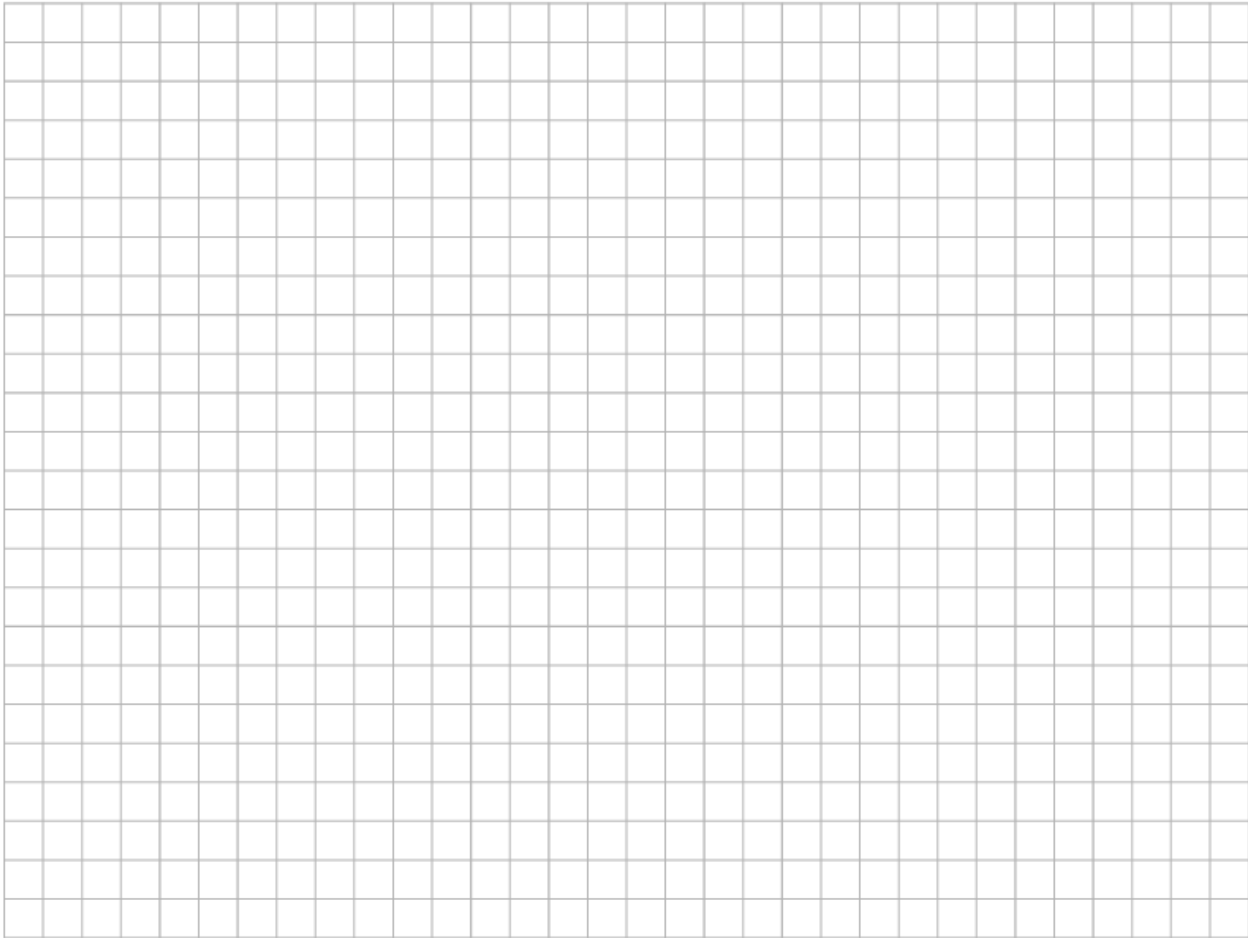
(a) Plot the four points on the diagram below.

(b) Describe two different ways of showing, using co-ordinate geometry techniques, that the points form a parallelogram $ABCD$.

First method:

Second method:

(c) Use one of the ways you have described to show that $ABCD$ is a parallelogram.

Question 17

Katie tossed a coin 200 times and threw 109 heads. Joe tossed the same coin 400 times and threw 238 heads. Lucy tossed the same coin 500 times and threw 291 heads. Katie, Joe and Lucy now think the coin may be biased.

(a) Give a reason why they think that the coin may be biased.

(b) Lucy uses all the above data and calculates that the best estimate of the probability of throwing a head with this coin is 0·58. Show how Lucy might have calculated this probability.

(c) Joe agrees with Lucy's estimate of 0·58 as the probability of throwing a head with this coin. He claims that the probability of throwing 3 successive heads with this coin is **less** than the probability of throwing 2 successive tails. Calculate the probability of each event and state whether Joe's claim is true or not.

Question 18

An unbiased circular spinner has a movable pointer and five equal sectors, two coloured green and three coloured red.
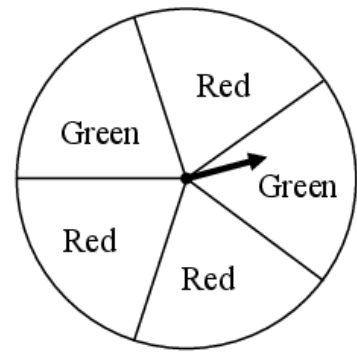
(a) **(i)** Find the probability that the pointer stops on green for one spin of the spinner.

(ii) List all the possible outcomes of 3 successive spins of the spinner.

(b) A game consists of spinning the spinner 3 times. Each time the spinner stops on green the player wins €1; otherwise the player wins nothing. For example, if the outcome of one game is "green, red, green" the player wins €2.

Complete the following table:

Player wins	€0	€1	€2	€3
Required outcomes				

(c) Is one spin of the spinner above an example of a Bernoulli trial?

Answer: _____

Explain what a Bernoulli trial is.

Question 19

(a) l is the line $3x + 2y + 18 = 0$. Find the slope of l.

(b) The line k is perpendicular to l and cuts the x-axis at the point $(7, 0)$.
Find the equation of k.

(c) Find the co-ordinates of the point of intersection of the lines l and k.